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ON THE WEAK AXIOM OF REVEALED PREFERENCE WITHOUT DEMAND CONTINUITY ASSUMPTIONS

by

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Abstract

Abstract: We consider two versions of the weak axiom of revealed preference for demand functions which are not necessarily continuous, and study their relationship to preference maximizing behavior. In particular, we characterize demand functions satisfying these axioms and obtain some results concerning the existence of a nontransitive consumer.

1. Introduction

Recently the weak axiom of revealed preference has proved to be of some importance to the theory of aggregate demand. For it was shown by W.Hildenbrand (1983) that under some assumptions on the income distribution of a group of consumers the market demand satisfies the weak axiom if all individual demand functions have this property. In a first version of his paper he raised the question whether there is a representative consumer for such a group, i.e. a preference relation from which the aggregate demand function can be derived.

Of course, this relation cannot be assumed to be transitive for it has been shown by J.Lenninghaus (1984) that in general such a market demand function does not satisfy the strong axiom of revealed preference. Therefore Hildenbrand's question is related to a conjecture of R.Kihlstrom, A.Mas-Colell, H.Sonnenschein, and W.Shafer (1976), which claims the existence of a representing nontransitive consumer in the sense of W.Shafer (1974) for continuous demand functions satisfying the axiom. For the special case in Hildenbrand's paper W.Shafer has given an essentially positive answer.

Our concern here is the "noncontinuous" nontransitive consumer, i.e. the purpose of this paper is to study the relationship between preference maximizing behavior and demand functions which satisfy the weak axiom but are not necessarily continuous.

We examine two versions of this axiom. The first one which we shall call the "weak axiom" seems to be the weakest form known from the literature. The second one is Samuelson's notion and, like in Hildenbrand (1983), denoted as "axiom".

After having given the basic definitions in the next section, we shall study the weak version in Section 3. Although we are primarily interested in demand functions it turns out that in this case we can generalize the results to demand correspondences at no additional cost. Roughly speaking, we can characterize demand functions (resp. correspondences) satisfying the weak axiom as selections of a demand correspondence which is derived from a preference relation with nice properties. By "nice" we mean that the preference relation should fulfill conditions which guarantee that nonempty, compact, and convex sets have best elements. In particular, they should imply the existence of a derived demand correspondence defined for all budget sets. In the context of the weak version without continuity, this seems to be a natural requirement in accordance with the idea which is behind Shafer's definition of a nontransitive consumer.

The axiom of revealed preference will be considered in Section 4. Since this notion implies unique demand, a "nice" preference relation in that case should additionally ensure the uniqueness of best elements in convex sets. The first theorem in this section characterizes demand functions satisfying the axiom as derived from preferences which do not have the required properties. But a second theorem shows that some further assumptions on the demand function imply that it can be derived from such a nice preference at least on its range.

Finally, the paper is finished by some concluding remarks in Section 5.

2. Definitions

Let $\mathbb{R}_+^\ell$ be the nonnegative orthant of the ℓ-dimensional Euclidean space $\mathbb{R}^\ell$ and define $P=\{x\in \mathbb{R}_+^\ell \mid 0\ll x\}$ [1]. For each $p\in P$, $B(p)=\{x\in \mathbb{R}_+^\ell \mid px\leq 1\}$ is the budget set with respect to the vector p of price-income ratios.

A *demand function (correspondence)* assigns to each budget set an element (a subset) of it, i.e. it is a function (corres-

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1) $a\ll b$ means $a_h<b_h$ for $h=1,\ldots,\ell$

pondence) $f: P \to \mathbb{R}_+^\ell$ such that

$pf(p) \leq 1$ for all $p \in P$.[2)]

A demand function f satisfies the *weak axiom (of revealed preference)* iff for all $p, q \in P$

$pf(q) < 1$ implies $qf(p) \geq 1$.

f satisfies the *axiom (of revealed preference)* iff for all $p, q \in P$

$pf(q) \leq 1$ and $qf(p) \leq 1$ implies $f(p) = f(q)$.

Observe that the weak axiom implies the budget identity $pf(p) = 1$ for all p. Therefore it does not follow from the axiom if this identity is not assumed.

A demand correspondence f satisfies the (weak) axiom iff each selection of f has this property. Of course, the validity of the axiom implies single-valued demand, i.e. f can be considered as a function.

Now let X be a subset of $\mathbb{R}_+^\ell$.

A binary relation R defined on X is called a *preference (relation) on* X. If $(x,y) \in R$ we call x as least as good as y and denote it equivalently by xRy or $x \in R(y)$.

A preference relation R on X is called

upper continuous, iff $R(x)$ is closed in X,

convex, iff $R(x)$ is convex,

weakly monotone, iff $x \ll y$ implies $x \notin R(y)$,

monotone, iff $x < y$ implies $x \notin R(y)$,

complete, iff $x \in R(y)$ or $y \in R(x)$,

antisymmetric, iff $x \in R(y)$ and $y \in R(x)$ implies $x = y$,

asymmetric, iff $x \in R(y)$ implies $y \notin R(x)$,

where each condition holds for all $x, y \in X$.

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2) Inequality for sets is defined pointwise

Let R be a preference relation on X.

If $Y \subseteq X$ then $y \in Y$ is an *R-best element in* Y iff

$y \in R(x)$ for all $x \in Y$.

If $p \in P$ then $f_R(p)$ denotes the set of all R-best elements in $B^X(p) := \{x \in X | px \leq 1\}$.

A demand function (correspondence) f is called *derived from* R *on* X, if $\{f(p)\} = f_R(p)$ $(f(p) = f_R(p))$, and *weakly derived*, if $f(p) \in f_R(p)$ $(f(p) \subseteq f_R(p))$, for all $p \in P$.

The revealed preference relation V of a demand correspondence (function) f is defined by

xVy iff there is $p \in P$ such that $x \in f(p)$ and $py \leq 1$.

3. Demand correspondences satisfying the weak axiom

In this section we shall characterize demand correspondences (and, therefore, demand functions) satisfying the weak axiom as weakly derived from preference relations with certain properties.

A fundamental step towards this goal is provided by the following result of H.Sonnenschein (1971).

Lemma 1: Let R be a relation on $X \subseteq \mathbb{R}^\ell$ with the following properties:

(i) R is upper continuous.

(ii) $\operatorname{conv} F \subseteq \bigcup_{x \in F} R(x)$ for all finite subsets F of X.[3)]

Then every nonempty, compact, and convex subset of X has an R-best element.

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3) conv S denotes the convex hull of a set S.

<u>Proof</u>: For the sake of completeness we repeat the main argument of H.Sonnenschein (1971).

Let C be a nonempty, compact, and convex subet of X. Consider the collection $\{R(x) \cap C \mid x \in C\}$ of subsets of C, which is nonempty because C is nonempty. Since $R(x)$ is closed, every set $R(x) \cap C$ is compact and therefore closed in $\mathbb{R}^{\ell}$. Take any finite collection $(R(x_i) \cap C)_{i=1}^{n}$ and any subset $F \subseteq \{x_1, \ldots, x_n\}$. Since C is convex, $\operatorname{conv} F \subseteq \bigcup_{x \in F} R(x) \cap C = \bigcup_{x \in F} (R(x) \cap C)$. By a corrollary to the Knaster-Kuratowski-Mazurkiewicz Lemma (see Sonnenschein (1971), Corollary 5) it follows that $\bigcap_{i=1}^{n} (R(x_i) \cap C) \neq \emptyset$. Therefore, $\{R(x) \cap C \mid x \in C\}$ has the finite intersection property which implies $\bigcap_{x \in C} R(x) \cap C \neq \emptyset$.

Q.E.D.

The next result gives a sufficient condition for a preference relation having a derived demand correspondence which satisfies the weak axiom.

<u>Proposition 1</u>: If R is a relation on $\mathbb{R}_+^{\ell}$ such that for all $x, y \in \mathbb{R}_+^{\ell}$

$$x \ll y \text{ implies } x \notin \operatorname{conv} R(y),$$

then any demand correspondence weakly derived from R satisfies the weak axiom.

<u>Proof</u>: For $p, q \in P$ let $x \in f_R(p)$, $y \in f_R(q)$, and $py < 1$. Assume that $qx < 1$. Since $px \leq 1$ and $py \leq 1$, we obtain $p(\alpha x + (1-\alpha) y) < 1$ and $q(\alpha x + (1-\alpha) < 1$ for $0 < \alpha < 1$. It follows the existence of $z \in B(p) \cap B(q)$ such that $\alpha x + (1-\alpha) y \ll z$. Since $x, y \in R(z)$, we get $\alpha x + (1-\alpha) y \in \operatorname{conv} R(z)$ which contradicts the assumption. Therefore, $qx \geq 1$.

Q.E.D.

In the sequel, we will prove the converse of this propostion. For that we need the following

Lemma 2: Let f be a demand correspondence which satisfies the weak axiom. If $p_1,\ldots,p_n \in P$, then

$$\mathrm{conv}(p_1,\ldots,p_n) \subseteq \bigcup_{i=1}^{n} S(p_i),$$

where $S(p_i) := \{q \in \mathbb{R}_+^{\ell} \mid qf(p_i) \geq 1\}$.

Proof: Let $p \in \mathrm{conv}(p_1,\ldots,p_n)$, i.e. $p = \sum_{i=1}^{n} \alpha_i p_i$, $\alpha_i \geq 0$, $\alpha_i = 1$. Assume that $p \notin \bigcup_{i=1}^{n} S(p_i)$, i.e. for each i there is $x_i \in f(p_i)$ with $px_i < 1$. This implies the existence of $q >> p$ such that $qx_i < 1$ for all i. Since f satisfies the weak axiom, we get $p_i f(q) \geq 1$ for all i. Take any $y \in f(q)$. Then $1 \leq \sum_{i=1}^{n} \alpha_i p_i y = py < qy$ which contradicts $qf(q) \leq 1$.
Therefore $p \in \bigcup_{i=1}^{n} S(p_i)$.

Q.E.D.

Using this result we can show that for a demand correspondence satisfying the weak axiom each set of elements revealed preferred to $x \in \mathbb{R}_+^{\ell}$ can be separated from x by a hyperplane with positive normal vector. Since this is trivial for $x \notin P$, we state this result as follows:

Proposition 2: Let f be a demand correspondence satisfying the weak axiom. Then for each $x \in P$ there exists $s \in \mathbb{R}_+^{\ell}$ such that

(a) $sx \leq 1$

(b) $sf(p) \geq 1$ for each $p \in P$ with $px \leq 1$.

Proof: Define a relation S on $\mathbb{R}_+^{\ell}$ by $S(p) = \{q \in \mathbb{R}_+^{\ell} \mid p \in P \Rightarrow qf(p) \geq 1\}$. If $x \in P$, then any S-best element s in $B(x)$ has the properties (a) and (b): $s \in B(x)$ implies $sx \leq 1$ and for $p \in P$ with $px \leq 1$, i.e. $p \in B(x)$, we know that $s \in S(p)$, i.e. $sf(p) \geq 1$.

Since for any $x \in P$ the set $B(x)$ is nonempty, compact, and convex, we can prove the proposition by applying Lemma 1 to the relation S, i.e. it remains to show the properties (i) and (ii) in Lemma 1 for S.

By definition of S, it is obvious that $S(p)$ is closed for all p.

Now let F be any finite subset of $\mathbb{R}^{\ell}_{+}$. If there is $q \in F \setminus P$ then $\bigcup_{p \in F} S(p) \supseteq S(q) = \mathbb{R}^{\ell}_{+}$ and we have $\operatorname{conv} F \subseteq \bigcup_{p \in F} S(p)$. Otherwise $F \subseteq P$ and we obtain this inclusion by direct application of Lemma 2. Q.E.D.

A straightforward consequence of Propositions 1 and 2 is a characterization of demand correspondences satisfying the weak axiom.

<u>Theorem 1</u>: Let f be a demand correspondence. Then the following conditions are equivalent:

(1) f satisfies the weak axiom.

(2) The revealed preference relation V has the property: $y \ll x$ implies $y \notin \operatorname{conv} V(x)$ for all $x, y \in \mathbb{R}^{\ell}_{+}$.

(3) f is weakly derived from a convex and weakly monotone preference relation on $\mathbb{R}^{\ell}_{+}$.

<u>Proof</u>: (1) $\Rightarrow$ (2): If $y \ll x$ then $x \in P$ and, by Proposition 2, there is $s \in \mathbb{R}^{\ell}_{+}$ such that $sx \leq 1$ and $sV(x) = s \bigcup_{px \leq 1} f(p) \geq 1$. This implies $sy < 1$ and $s \operatorname{conv} V(x) \geq 1$, i.e. $y \notin \operatorname{conv} V(x)$.

(2) $\Rightarrow$ (1): Define a relation R on $\mathbb{R}^{\ell}_{+}$ by $R(x) = \operatorname{conv} V(x)$. R is convex by definition, (2) states that R is weakly monotone, and $V \subseteq R$ trivially implies that f is weakly derived from R.

(3) $\Rightarrow$ (1): Proposition 1. Q.E.D.

A weak point of the theorem is the fact that in general the preference relations in the characterizing class do not have nice properties. A convex and weakly monotone relation does not ensure the existence of a derived demand correspondence. One would like to characterize demand correspondences satisfying the weak axiom by a class of preferences which guarantee that at least any nonempty, compact and convex set has best elements. The next result shows that this is possible. Moreover, by applying a duality argument to Proposition 1 and Lemma 2 we can derive the demand correspondences from preferences R where the sets $R(x)$ are closed convex cones with vertex x which do not contain any $y<<x$.

<u>Theorem 2</u>: If f is a demand correspondence satisfying the weak axiom, then f is weakly derived from a preference relation R on $\mathbb{R}^{\ell}_{+}$ with the following properties:

1) R is upper continuous and convex.

2) R is weakly monotone.

3) $\operatorname{conv} F \subseteq \bigcup_{x \in F} R(x)$ for any finite $F \subseteq \mathbb{R}^{\ell}_{+}$.

4) R is complete.

More specifically, R can be chosen such that for $x \in \mathbb{R}^{\ell}_{+}$

$$R(x)=\{y\in \mathbb{R}^{\ell}_{+} \mid x\in P \Rightarrow s(x)y\geq 1\},$$

where $s(x)$ is a (dual) demand correspondence which satisfies the weak axiom.

<u>Proof</u>: If f is weakly derived from a relation R which is defined by a correspondence s as stated above, then $s:P\to\mathbb{R}^{\ell}_{+}$ must have the property that $x\in B(p)\cap P$ implies $f(p)\subseteq R(x)$, i.e. $s(x)f(p)\geq 1$. This means that for any $s\in s(x)$ the inequality

$sf(p) \geq 1$ holds for each $p \in P$ with $px \leq 1$. Therefore it is necessary to define $s(x)$ to be a subset of all s which satisfy conditions (a) and (b) of Proposition 2.

We choose for s any such correspondence whose existence is guaranteed by Proposition 2.

Since s is a (dual) demand correspondence which is weakly derived from the relation S defined in the proof of that proposition, s satisfies the weak axiom by (the dual of) Proposition 1 if we can show that $q \ll p$ implies $q \notin \text{conv } S(p)$ for all $p, q \in \mathbb{R}^{\ell}_{+}$.

Observe first that $S(p)$ is convex by definition. If $q \ll p$ then $p \in P$ and for any $x \in f(p)$ we obtain $1 = px > qx$, i.e. $q \notin S(p)$.

We have proved that s is a demand correspondence satisfying the weak axiom and that f is weakly derived from R defined by $R(x) = \{y \in \mathbb{R}^{\ell}_{+} \mid x \in P \Rightarrow s(x)y \geq 1\}$. It remains to verify the properties 1) to 4).

1) follows immediately from the definition of R. To prove 2), let $y \ll x$. This implies $x \in P$ and $1 = sx > sy$ for any $s \in s(x)$, i.e. $y \notin R(x)$.

Now let F be any finite subset of $\mathbb{R}^{\ell}_{+}$. If there is $y \in F \setminus P$ we get $\text{conv } F \subseteq R(y) = \bigcup_{x \in F} R(x)$. If $F \subseteq P$ we obtain 3) by application of Lemma 2 to the correspondence s.

Finally, the completeness of R is an immediate consequence of the weak axiom for the correspondence s. Q.E.D.

We remark that the largest weakly monotone relation M (defined by $x \notin M(y)$ iff $x \ll y$) has all the properties mentioned in Theorem 2 except for convexity. Since any demand function satisfying the budget identity is weakly derived from M the weak version should not be interpreted as a property of consistent behavior but as a consequence of rationality described by convex preferences.

4. Demand functions satisfying the axiom

The first result of this section characterizes demand functions which satisfy the axiom of revealed preference. Its proof is straightforward and the main argument can already be found in the work of M.K.Richter (1966,1971).

Theorem 3: Let f be a demand function. Then the following conditions are equivalent:

(1) f satisfies the axiom.

(2) The revealed preference relation V is antisymmetric.

(3) f is weakly derived from an antisymmetric relation on $\mathbb{R}_+^\ell$.

(4) f is derived from an antisymmetric relation on $\mathbb{R}_+^\ell$.

(5) There is an asymmetric relation Q on $\mathbb{R}_+^\ell$ which rationalizes f in the sense that for all $p \in P$, $x \in \mathbb{R}_+^\ell$:

(i) $xQf(p)$ implies $px>1$.

(ii) $px \leq 1$ and $x \neq f(p)$ implies $f(p)Qx$.

Proof: (1) $\Rightarrow$ (2): xVy and yVx imply the existence of $p,q \in P$ such that $x=f(p)$, $y=f(q)$, $py \leq 1$, and $qx \leq 1$. From the weak axiom it follows $f(p)=f(q)$, i.e. $x=y$.

(2) $\Rightarrow$ (3): Since $f(p)Vx$ for all $x \in B(p)$, f is weakly derived from V.

(3) $\Rightarrow$ (4): If R is antisymmetric, there can be at most one

R-best element in any set. Since f is weakly derived from R, $f(p)$ is an R-best element in $B(p)$. Therefore it is the only one, i.e. f is derived from R.

(4) ➡ (5): Define $x \in Q(y)$ iff $x \in R(y)$ and $y \notin R(x)$. Q is asymmetric since $x \in Q(y)$ implies $y \notin R(x)$, i.e. $y \notin Q(x)$.

To prove (i), assume $x \in Q(f(p))$. By definition of Q, $f(p) \notin R(x)$. Since f is derived from R, $f(p) \in R(y)$ for $y \in B(p)$. Therefore $x \notin B(p)$, i.e. $px > 1$.

To prove (ii), assume $px \leq 1$ and $x \neq f(p)$. Since f is derived from R, $f(p) \in R(x)$. Antisymmetry of R implies $x \notin R(f(p))$. By definition of Q, we get $f(p) \in Q(x)$.

(5) ➡ (1): Let Q be a relation satisfying (i) and (ii) and assume $pf(q) \leq 1$ and $qf(p) \leq 1$. (i) implies $f(q) \notin Q(f(p))$. By (ii), it follows $f(p) = f(q)$. Q.E.D.

This characterization theorem is not fully satisfactory for the criticism of Theorem 1 applies here too: An antisymmetric relation does not guarantee the existence of a derived demand function. Of course, if we would have a demand function f such that the budget identity $pf(p)=1$ holds for each $p \in P$ then the axiom implies the weak axiom and we could apply Theorem 2. But that would not ensure unique best elements in each budget set which in this case would be desirable.

Even if the smallest relation R within the class described in Theorem 2 is chosen, the following example shows that in general demand functions different from f can be derived from R (this would be the case even if R is restricted to the range of f, as a slight modification of the example can demonstrate).

Example: Consider the demand function $f:P \to \mathbb{R}_+^2$ which is defined by

$$f(p_1,p_2) = \begin{cases} (\frac{1}{p_1}, 0) & \text{if } p_1<1. \\ (0, \frac{1}{p_2}) & \text{if } p_1\geq 1. \end{cases}$$

The axiom can be easily verified and the smallest relation of the form described in Theorem 2 is given by

$y \in R(x)$ iff $x \in P$, $sx \leq 1 \leq sV(x)$ implies $sy \geq 1$.

Straightforward checking (Fig. 1) shows that for $x=(x_1,x_2)$ with $0<x_1<1$ and $x_2>0$ there is only one $s \in \mathbb{R}_+^2$ satisfying the inequality $sx \leq 1 \leq sV(x)$. It is normal to the straight line through x and $(1,0)$, i.e. $s=(1, (1-x_1)/x_2)$.

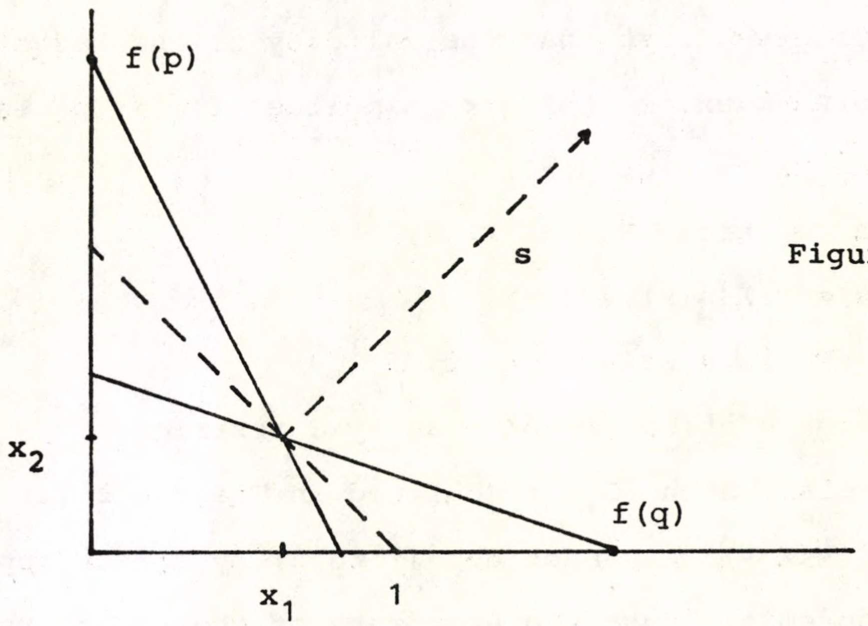

Figure 1

If p is chosen such that $p_1=1$, then no point on the budget line given by p is below the straight line through $(1,0)$ and any $x \in P$ with $px \leq 1$. It follows that all points on the budget line are R-best elements in $B(p)$.

Nevertheless there is a positive result. For if the range X of the demand function is convex there exists a preference relation R on X such that each nonempty, compact and convex subset of X has a unique R-best element.

Theorem 4: Let f be a demand function with convex range X such that $pf(p)=1$ for all $p \in P$.

If f satisfies the axiom, then f is derived from a preference relation R on X with the following properties:

(1) R is upper continuous and convex.

(2) $\operatorname{conv} F \subseteq \bigcup_{x \in F} R(x)$ for any finite $F \subseteq X$.

(3) R is complete.

(4) R is monotone.

(5) $x,y \in R(z)$ for all $z \in [x,y]$ implies $x=y$.

Proof: Observe first that the validity of the budget identity implies the weak axiom ($pf(q)<1$ implies $f(q) \neq f(p)$ and therefore, by the axiom, $qf(p)>1$).

Define a relation R on X by

$$R(x)=\{y \in X \mid s(x)y \geq 1\},$$

where $s: X \to P$ is a selection of f^{-1}.

Since this relation is not exactly a restriction of a relation on $\mathbb{R}_+^\ell$ as defined in Theorem 2 (it is different for $x \in X \setminus P$) we cannot get 1) to 3) by direct application of this theorem although the arguments of the proof are essentially the same.

1) follows immediately from the definition.

To prove 2), let $x_1,\dots,x_n \in X$, $x=\sum_{i=1}^{n} \alpha_i x_i$, $\alpha_i \geq 0$, $\Sigma\alpha_i=1$ and assume $x \notin R(x_i)$ for all i. This implies $s(x_i)x<1$ and, by the axiom, $s(x)x_i>1$ for all i. Summing up, we obtain $s(x)x=\sum_{i=1}^{n} \alpha_i s(x)x_i>1$, which is a contradiction to the budget identity. Therefore, $x \in \bigcup_{i=1}^{n} R(x_i)$.

If $y \notin R(x)$ then $s(x)y<1$. The axiom implies $s(y)x>1$, i.e. $x \in R(y)$. By definition, this means that R is complete.

To prove 4), let $x,y \in X$ such that $y<x$. $s(x) \in P$ implies $s(x)y<s(x)x=1$, i.e. $y \notin R(x)$.

Assume now $x,y \in R(z)$ for all $z \in [x,y]$ and $x \neq y$. Choose two points $x',y' \in]x,y[$ such that $x' \neq y'$. Since $x \in R(x')$ and $y \in R(x')$, it follows $s(x')x \geq 1$ and $s(x')y \geq 1$. But $x' \in]x,y[$ and $s(x')x'=1$ implies $s(x')x=s(x')y=1$, i.e. $s(x')[x,y]=1$ and therefore $s(x')y'=1$.

By the same argument we obtain $s(y')x'=1$. The axiom implies $x'=y'$ which contradicts the assumption $x' \neq y'$, i.e. we have proved 5).

It remains to show that f is derived from R on X, i.e. that $f(p)$ is the unique R-best element in $B^X(p)$ for each p.

If $x \in X$ such that $px \leq 1$ and $s(x)f(p) \leq 1$, the axiom implies $x=f(p)$, i.e. $s(x)f(p)=1$. Therefore $s(x)f(p) \geq 1$ for all $x \in B^X(p)$, i.e. $f(p)$ is an R-best element in $B^X(p)$.

It follows from 5) that $f(p)$ is the unique R-best element in $B^X(p)$. Q.E.D.

This theorem corresponds to an analogous result for demand functions satisfying the strong axiom of revealed preference which is stated as Theorem 1 in L.Hurwicz and M.K.Richter (1971), and where a convexity assumption on the range is also needed.

5. Concluding remarks

While the examination of the weak axiom in Section 3 has led to a satisfactory characterization we cannot claim the same for the results of Section 4. Theorem 3 characterizes demand functions satisfying the axiom essentially by properties of the revealed preference relation which is too small to meet the desired requirements discussed above. To put it differently, this theorem is almost a reformulation of the definition.

On the other hand, the weakness of Theorem 4 is that it only proves the existence of a nice representing preference relation on a subset of $\mathbb{R}^{\ell}_{+}$. Moreover we need a convexity assumption on the range of the demand function.

Nevertheless, the result is related to the conjecture of Kihlstrom, Mas-Colell, Sonnenschein, and Shafer (1976). This conjecture claims that continuous demand functions satisfying the axiom can be represented by a nontransitive consumer in the sense of Shafer (1974), i.e. they are derived from a complete, continuous and strongly convex preference on $\mathbb{R}^{\ell}_{+}$.

By Theorem 4, we have almost shown the corresponding statement for the noncontinuous case if the demand function is surjective. The representing preference is complete, upper continuous and has some properties comparable to strong convexity (of course, it cannot be continuous since this would imply continuous demand which we do not assume).

In other words, if a demand function with convex range X satisfies the axiom, it can be represented by some kind of a noncontinuous nontransitive consumer with consumption set X.

Modifications of the example given in the previous section indicate that one cannot hope to get much better results without assuming continuity. But avoiding the convexity assumption and obtaining a representing preference on the whole set $\mathbb{R}^{\ell}_{+}$ seems to be possible for the continuous case which we shall examine in a subsequent paper.

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